SESSIONAL EXAM-2022
SUB: MATHEMATICS
CLASS: THIRD Semester
Paper MAT-**HC**-3026 (Group Theory)
Time-60 min
Marks-20

Answer any four questions: 4×5

1. Show that the subset $\{1, -1, i, -i\}$ of complex numbers form a group under complex multiplication.

2. Show that GL(2,$\mathbb{R}$) defined by =

$$GL(2,\mathbb{R}) = \left\{ \begin{bmatrix} a & b \\ c & d \end{bmatrix} : a, b, c, d \in \mathbb{R}, ad - bc \neq 0 \right\}$$

   is a non-Abelian group under matrix multiplication.

3. Let U(n)be the set of all positive integers less than n and relatively prime to n. Show that U(15) is a group under the operation multiplication modulo 15.

4. With diagrams and words, describe each symmetry in D_4.(the set of Symmetries of a Square).

5. Define binary operation, group and $SL(2,\mathbb{R})$ group

6. Let G be a group and H be a subgroup of G, and let a,b are in G. Show that (i) $a \in aH$ and (ii) $aH = bH$ if and only if $ab^{-1} \in H$.

SESSIONAL EXAM-2022
SUB: MATHEMATICS
CLASS: First Semester
Paper-MAT-**HC**-1026 (Algebra)
Time-60 min, Marks-20

Answer any FOUR questions 4×5

1. Express $\frac{2+3i}{4-5i}$ in the form $x + iy$ and in polar form.

2. Let $\mathbb{Z}$ be the set of integers. Let $n \neq 0$ be any fixed integer.For any $a, b \in \mathbb{Z}$ define the relation $a \equiv b(modn)$ if and only if n divides $a - b$. Show that This is an equivalence relation.

3. Using mathematical induction show that $n! \geq 2^{n-1}$ for all integers $n \geq 1$

4. Find the general solution of the linear system of equations

$$\begin{aligned} x_1 - 2x_2 + x_3 &= 0 \\ 2x_2 - 8x_3 &= 0 \\ -4x_1 + 5x_2 + 9x_3 &= 0 \end{aligned}$$

5. Write the vector $v = (4, 9, 19)'$ as a linear combination of $u_1 = (1, -2, 3)'$,$u_2 = (3, -7, 10)'$ and $u_3 = (2, 1, 9)'$

6. Apply elementary row operation to test the consistency of the system of equations

$$\begin{aligned} x_2 - 4x_3 &= 8 \\ 2x_1 - 3x_2 + 2x_3 &= 1 \\ 5x_1 - 8x_2 + 7x_3 &= 1 \end{aligned}$$

SESSIONAL EXAM-2022
SUB: MATHEMATICS
CLASS: FIFTH SEMESTER (Hons)
Paper-MAT-**HE**-5116 (Number Theory)
Time-60 min, Marks-20

Answer any four questions 4×5

1. Find $gcd(256, 1166)$. Hence show that the $gcd(256, 1166)$ can be expressed as a linear combination of 256 and 1166.

2. Find all positive solutions of $16x + 27y = 390$

3. Show that the linear Diophantine equation $ax + by = c$ has a solution if and only if d/c, where $d = gcd(a, b)$.
   If x_0, y_0 is any particular solutions of this equation, then all other solutions are given by $x = x_0 + \frac{b}{d}t$ and $y = y_0 - \frac{a}{d}t$ where t is an arbitrary integer.

4. Show that every positive integers can be expressed as a product of times.

5. Prove that there are infinite number of primes.

6. For arbitrary integers a and b, $a \equiv b(modn)$ if and only if a and b leave the same non-negative remainder when divided by n
   Let $n > 1$ be fixed and a,b,c,d be any integers. Then show that the following properties hold:
   (a) If $a \equiv b(modn)$ then $b \equiv a(modn)$
   (b) If $a \equiv b(modn)$ and $b \equiv c(modn)$ then $a \equiv c(modn)$

7. Solve the simultaneous system of linear equations by Chinese Remainder Theorem

$$x \equiv 2(mod3),$$
$$x \equiv 3(mod5),$$
$$x \equiv 2(mod7)$$

8. Find $\phi(210), \tau(210)$ and $\sigma(210)$

SESSIONAL EXAM-2022
SUB: MATHEMATICS
CLASS: First Semester (Regular)
Paper-MAT-**RC**-1016(Calculus)
Time-60 min, Marks-20

Answer any FOUR questions 4×5

1. Find limit (Any two)
   (i) $\lim\limits_{x \to -1} \frac{x^2+3x+2}{x^2-x2}$

   (ii) $\lim\limits_{x \to 0} \frac{1-cosx}{x^2}$

   (iii) $\lim\limits_{x \to \frac{\pi}{4}} (1 - tanx)sec2x$

2. Discuss the continuity of the function

$$f(x) = \begin{cases} x^2 & \text{if } x \leq 0, \\ 1 - x & x > 0 \end{cases}$$

3. Find the value of a and b if the function

$$f(x) = \begin{cases} 3ax + b & \text{if } x > 1, \\ 11 & \text{if } x = 1, \\ 5ax - 2b & x < 1 \end{cases}$$

   is continuous at $x = 1$

4. Discuss the differentiability of the function

$$f(x) = \begin{cases} 1 + x & \text{if } x \leq 2, \\ 5 - x & x > 2 \end{cases}$$

5. Discuss the differentiability of the function
   $f(x) = x|x|$, $x \in \mathbb{R}$ at $x = 0$

SESSIONAL EXAM-2022
SUB: MATHEMATICS
CLASS: FIFTH SEMESTER(REGULAR)
Paper-MAT-**RE**-5116 (Number Theory)
Time-60 min, Marks-20

Answer any four questions 4×5

1. Find $gcd(256, 1166)$. Hence show that the $gcd(256, 1166)$ can be expressed as a linear combination of 256 and 1166.

2. Find all positive solutions of $16x + 27y = 390$

3. Show that the linear Diophantine equation $ax + by = c$ has a solution if and only if d/c, where $d = gcd(a, b)$.
   If x_0, y_0 is any particular solutions of this equation, then all other solutions are given by $x = x_0 + \frac{b}{d}t$ and $y = y_0 - \frac{a}{d}t$ where t is an arbitrary integer.

4. Show that every positive integers can be expressed as a product of times.

5. Prove that there are infinite number of primes.

6. For arbitrary integers a and b, $a \equiv b(modn)$ if and only if a and b leave the same non-negative remainder when divided by n
   Let $n > 1$ be fixed and a,b,c,d be any integers. Then show that the following properties hold:
   (a) If $a \equiv b(modn)$ then $b \equiv a(modn)$
   (b) If $a \equiv b(modn)$ and $b \equiv c(modn)$ then $a \equiv c(modn)$

7. Solve the simultaneous system of linear equations by Chinese Remainder Theorem

$$x \equiv 2(mod3),$$
$$x \equiv 3(mod5),$$
$$x \equiv 2(mod7)$$

8. Find $\phi(210), \tau(210)$ and $\sigma(210)$

SESSIONAL (CBCS) EXAM-2021
SUB: MATHEMATICS
Class: FIFTH SEMESTER(H)
Paper-MAT-**HC**-5026 (Linear Algebra)
Time-60 Min, Marks-20

Answer any six questions: 4×5

1. (a) What is a vector space ?
   (b)Let V be a vector space and c be a scalar. Show that $0\mathbf{u} = 0$ and $-\mathbf{u} = (-1)\mathbf{u}$. Where $\mathbf{u}$ is a vector in V.

2. Let V be a vector space over the field F and U and W be two subspaces of V.Show that $W \cap W$ is again a subspace of V. Is $U \cup W$ a subspace of V ?

3. Let V be a vector space over the field F and let $\mathbf{v}_1$ and $\mathbf{v}_2$ be in V. Let $H = Span\{\mathbf{v}_1, \mathbf{v}_2\}$. Show that H is a subspace of V.

4. (a) Find a spanning set for the null space of the matrix

$$A = \begin{bmatrix} -3 & 6 & -1 & 1 & -7 \\ 1 & -2 & 2 & 3 & -1 \\ 2 & -4 & 5 & 8 & -4 \end{bmatrix}$$

   (b) Given $W = \left\{ \begin{bmatrix} 6a-b \\ a+b \\ -7a \end{bmatrix} : a, b \in \mathbb{R} \right\}$.Find a matrix A such that $W = ColA$.

5. (i)Find the eigenvalues of the matrix $A = \begin{bmatrix} 2 & 3 \\ 3 & -6 \end{bmatrix}$
   (ii) Given $\mathbf{u} = \begin{bmatrix} 2 \\ -5 \\ -1 \end{bmatrix}, \mathbf{v} = \begin{bmatrix} 3 \\ 2 \\ -3 \end{bmatrix}$. Compute $\mathbf{v}.\mathbf{u}$

6. Show that two vectors $\mathbf{v}$ and $\mathbf{u}$ in $\mathbb{R}^n$ are orthogonal to each other if $\mathbf{v}.\mathbf{u} = 0$. In such cases show that $\|\mathbf{u} + \mathbf{v}\|^2 = \|\mathbf{u}\|^2 + \|\mathbf{v}\|^2$

7. Define linear transformation T from a vector space V to another vector space W. Show that if is a subspace U of V, then T(U) is a subspace of W.

SESSIONAL EXAM-2022
SUB: MATHEMATICS
CLASS: FIFTH SEMESTER (REGULAR)
Paper-MAT-**RE**-5016 (COMPLEX ANALYSIS)
Time-60 min, Marks-20

Answer any four questions 4×5

1. What is meant by modulus and principal argument of a complex number? 2

2. Find the principal argument of $-4+4\sqrt{3}i$? 2

3. Define open ball and interior point. 2

4. Find the interior of $A = \left\{ z \in \mathbb{C} : |z| < 1 \right\}$ 3

5. Prove that $(o, 1)$ is open in $\mathbb{R}$ but not in $\mathbb{C}$ 4

6. Prove that $\left(A \bigcup B \right)^0 = A^0 \bigcap B^0$ 4

7. Check whether $A = \left\{ zin\mathbb{C} : |z| = 1, Argz = \frac{\pi}{n}, nin\mathbb{N} \right\}$ is open or closed.3

OR

8. Define limit point in $\mathbb{C}$. Show that $A = \left\{ z \in \mathbb{C} : |z| < 1 \right\}$ is open. 3