

2016

PHYSICS

(Major)

Paper : 5.1

Full Marks : 60

Time : 3 hours

*The figures in the margin indicate full marks
for the questions*

GROUP—A

(Mathematical Methods)

(Marks : 30)

1. Answer the following questions : 1×4=4

(a) Find the real part of $\frac{1+z}{1-z}$.

(b) What is the argument of $-3i$?

(c) Define pole and residue.

(d) Find the principal value of i^i .

2. (a) Find the complex conjugate of the functions

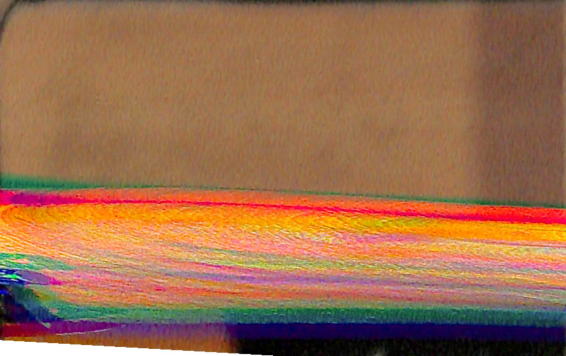
$$(x+iy) \cdot (a+ib) \text{ and } \frac{x-iy}{a+ib}$$

where x, y, a and b are real.

4

(b) Obtain the modulus of the complex number $\frac{1-i}{1+i}$.

2



(2)

3. (a) State De Moivre's theorem. 2
(b) Using De Moivre's formula, evaluate
 $(\cos 20^\circ + i \sin 20^\circ)^9$ 2

4. (a) Define equivalent contour. 2
(b) Verify if the function $f(z) = z$ is analytic. 4

Or

- (i) Determine if the function e^{iz} is analytic. 2

- (ii) Prove that

$$\left(\frac{z_1}{z_2} \right) = \arg z_1 - \arg z_2 \quad 2$$

5. (a) Check the analyticity and hence find the derivative of the function $f(z) = \sin z$. 5

Or

Find Taylor series expansion about the origin for $f(z) = \ln(1+z)$. 5

- (b) Find Laurent expansion for the function $f(z) = \frac{\sin z}{z^4}$ about $z_0 = 0$ and hence classify the singularity and calculate the residue. 5

Or

State and prove Cauchy's integral theorem. 5

(3)

GROUP—B

(Classical Mechanics)

(Marks : 30)

6. Answer the following questions : $1 \times 3 = 3$

- (a) State the principle of virtual work.
(b) Define central force and write down a general expression for it.
(c) Define Hamiltonian of a system.

7. Answer any *three* of the following questions : $2 \times 3 = 6$

- (a) Explain with example the meaning of holonomic constraint.
(b) Show that angular momentum is a constant of central force motion.
(c) In a two-body system, the masses are in the ratio 4:1. The mass of the lighter body is 10^{-28} g. Estimate the reduced mass of the system.
(d) What are generalized coordinates and generalized velocities?

8. Answer any *four* of the following questions : $4 \times 4 = 16$

- (a) Show that a two-body central force problem can be reduced to one-body problem.

- (b) Find the equation of motion of a system with the given Lagrangian

$$L = \frac{1}{2} e^{\alpha t} (\dot{x}^2 - \omega^2 x^2)$$

where α and ω are constants.

- (c) Obtain the general differential equation of a central orbit.
- (d) Show that if the Lagrangian function does not contain the time explicitly, the total energy of the conservative system is conserved.
- (e) Construct the Lagrangian and hence equation of motion of a simple pendulum placed in a uniform gravitational field.
- (f) Set up Lagrangian equation for an Atwood machine and find an expression for its acceleration.

9. Establish Hamilton's canonical equations. 5

Or

Obtain Lagrange's equation of motion for a conservative system using D'Alembert's principle. 5

