2017

PHYSICS

(Major)

Paper : 3.1

(Mathematical Methods-III and Electrostatics)

Full Marks : 60

Time : 3 hours

The figures in the margin indicate full marks for the questions

GROUP—A

(Mathematical Methods)

(*Marks* : 25)

1. Answer the following questions : 1×3=3

(a) What do you mean by an idempotent matrix?

(b) Show that

$$A = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$$

is a self-adjoint matrix.

(c) Is $S = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$ a symmetric matrix?

2. Show that

$$A = \begin{pmatrix} 1 & 1 & 3 \\ 5 & 2 & 6 \\ -2 & -1 & -3 \end{pmatrix}$$

is a nilpotent matrix. 2

3. Answer any *two* of the following questions : 5×2=10

(a) (i) Show that every square matrix can be expressed uniquely as the sum of a Hermitian and a skew-Hermitian matrix. 2

(ii) For the matrix $A = \begin{pmatrix} 1 & 2 \\ 3 & -5 \end{pmatrix}$, verify the theorem

$$A\,(\text{adj } A) = (\text{adj } A)\,A = |A|\,I$$ 3

(b) (i) Show that the trace of a product of two matrices is independent of the order of multiplication. 2

(ii) Determine the eigenvalues of the matrix $A = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$. 3

(c) (i) Prove that the product of a singular matrix with its adjoint is the null matrix. 2

(ii) What is the rank of the matrix

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ -3 & -6 & -9 \end{pmatrix}?$$ 3

4. Answer *either (a) and (b) or (c) and (d)* : 5×2=10

(a) (i) For three Pauli spin matrices $\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$, $\sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$ and $\sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$, show that $\sigma_1\sigma_2 = i\sigma_3$. 2

(ii) Prove that the modulus of each eigenvalue of a unitary matrix is unitary. 3

(b) Diagonalize the matrix

$$A = \begin{pmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{pmatrix}$$ 5

(c) State Cayley-Hamilton theorem. Obtain the eigenvalue equation of the matrix

$$A = \begin{pmatrix} 1 & 1 & 2 \\ 3 & 1 & 1 \\ 2 & 3 & 1 \end{pmatrix}$$

and verify the Cayley-Hamilton theorem. 5

(d) Using matrix, find the transformation equation for a 90 degree rotation about the z-axis of Cartesian coordinate system. 5

GROUP—B

(Electrostatics)

(*Marks* : 35)

5. Choose the correct answer/Answer the following questions : 1×4=4

(a) Potential at a large distance from a quadrupole is proportional to

(i) $\frac{1}{r}$ (ii) $\frac{1}{r^2}$

(iii) $\frac{1}{r^3}$ (iv) $\frac{1}{r^4}$

(b) How much electric energy is stored by a solid sphere of radius R and total charge Q?

(Continued)

(c) Electric field vector is

(i) rotational

(ii) irrotational

(d) Write Gauss' law of electrostatics in differential form.

6. Answer the following questions : 2×3=6

(a) Draw a schematic diagram of an electric octopole.

(b) Two charges 5 microcoulomb each are at the points (0, 3, 0) and (3, 0, 0) respectively. Find the electric intensity at (0, 0, 5).

(c) The electric field due to a short dipole at a point distant 1 cm from it on its perpendicular bisector is $1{\cdot}5 \times 10^{-11}$ volt/m. Find the dipole moment.

7. Using integral form of Gauss' law in electrostatics, find the electric field and potential at a distance z above the centre of a circular loop of radius R that carries a uniform line charge λ. 5

Or

Find an expression for the torque experienced by an electric dipole in external electric field. Hence show that the work done

(Turn Over)

in rotating the dipole from an initial position θ_1 to θ_2 is $W = -pE(\cos\theta_2 - \cos\theta_1)$. 3+2=5

8. Answer any *two* questions : 10×2=20

(a) *(i)* Using Gauss' law, find the electric field outside a uniformly charged solid sphere of radius R and total charge q. 3

(ii) Compute the curl and divergence of electric field given by

$$\vec{E}(x, y, z) = (x^2 + z^2 + 5)\hat{i} + (x^2 + y^2 - 9z)\hat{j} + (y^2 + z^2)\hat{k}$$ 2

(iii) Establish the Clausius-Mosotti equation for a linear dielectric material. 5

(b) *(i)* Define electric susceptibility, permeability and dielectric constant of material. 3

(ii) Using Laplace's equation, obtain an expression for potential inside a spherical capacitor of two concentric spheres having inner radius a and outer radius b. 3

(iii) State and prove uniqueness theorem. 4

(c) *(i)* An isotropic dielectric is placed in an uniform electrostatic field E. Show that the field inside a spherical cavity in this direction is

$$E_i = E + \frac{P}{3\varepsilon_0}$$

where P is the polarization. 5

(ii) Write Poisson's equation. Solve Laplace's equation to find the potential at a distance r from the axis of an infinitely long conducting cylinder of radius R charged with a surface density σ. 1+4=5

(d) *(i)* A point charge is situated near an infinite plane earthed conductor. Apply the method of electrical image to calculate—

1. the surface charge density induced on the plane;
2. the force between the plane and the charge. 4+3=7

(ii) An electron is at a distance 10 Å from an infinite plane conductor. Calculate the force experienced by the electron and the work done in moving it to infinite distance away from the conductor. 3

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