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3 (Sem-5/CBCS) PHY HE3

2021

(Held in 2022)

PHYSICS

(Honours Elective)

Paper : PHY-HE-5036

(Advanced Mathematical Physics)

Full Marks : 60

Time : Three hours

The figures in the margin indicate full marks for the questions.

1. Answer the following questions : $1 \times 7 = 7$
- (a) Define the linear vector space.
 - (b) What is field ? Give *two* examples.
 - (c) Show that $(A + B)(A - B) = A^2 - B^2$ if and only if A and B commute.

Contd.

- (d) Define quotient law of tensors.
- (e) Give the transformation equation for the tensor A_{st}^{lmn} and write its rank.
- (f) What is Minkowski space?
- (g) What do you mean by basis in a linear vector space?

2. Answer the following questions : $2 \times 4 = 8$

- (a) if $\vec{x}$, $\vec{y}$ and $\vec{z}$ are linearly independent vectors, determine whether $\vec{x} + \vec{y}$, $\vec{y} + \vec{z}$ and $\vec{z} + \vec{x}$ are linearly dependent or independent.
- (b) In a vector space $V(F)$, for all $v \in V$ and $f \in F$, prove that
 - (i) $\theta = 0.v$
 - (ii) $(-f)v = f(-v) = -(fv)$

(c) Diagonalize the matrix $\begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix}$.

(d) Show that $\text{div}(\text{curl } \vec{F}) = 0$ for a vector $\vec{F}$ using tensor analysis.

3. Answer **any three** of the following questions : $5 \times 3 = 15$

- (a) Find whether the set of vectors $[x_1, y_1, z_1]$ in R^3 , such that $x_1 + y_1 + z_1 = 0$ forms a subspace of R^3 .
- (b) Show that every linearly independent vector belonging to a vector space has a unique representation as a linear combination of its base vector.

- (c) Find the vector associated with the given second-order antisymmetric tensor

$$\begin{bmatrix} 0 & (x+y+z) & -(x+y) \\ -(x+y+z) & 0 & x \\ (x+y) & -x & 0 \end{bmatrix}$$

- (d) Define Kronecker delta. Show that it is a mixed tensor of rank 2.

- (e) What is alternating tensor? Prove that

$$\varepsilon_{iks} \varepsilon_{mps} = \delta_{im} \delta_{kp} - \delta_{ip} \delta_{km} = 0.$$

4. Answer **any three** of the following questions : 10×3=30

- (a) (i) Prove that the set I of all the integers with the binary operation * defined by $a * b = a + b - 1$ form a group. 4

- (ii) Determine e^A , when $A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$. 6

- (b) (i) Verify Cayley-Hamilton theorem

for the matrix $\begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix}$. Hence

find A^{-1} .

3+1=4

- (ii) Solve the coupled homogeneous linear differential equation using matrix method

$$\frac{dy_1}{dt} = 3y_1 + y_2$$

$$\frac{dy_2}{dt} = y_1 + 3y_2$$

subject to initial conditions

$$y_1(0) = 3; y_2(0) = 5.$$

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- (c) Using tensors, prove the following identities : 3+3+4=10

(i) $\bar{\nabla} \times (\phi \bar{A}) = \phi (\bar{\nabla} \times \bar{A}) + (\bar{\nabla} \phi) \times \bar{A}$

$$(ii) \quad \vec{\nabla} \times (\vec{\nabla} \times \vec{A}) = \vec{\nabla}(\vec{\nabla} \cdot \vec{A}) - \nabla^2 \vec{A}$$

$$(iii) \quad \vec{\nabla} \times (\vec{A} \times \vec{B}) = \vec{A}(\vec{\nabla} \cdot \vec{B}) - \vec{B}(\vec{\nabla} \cdot \vec{A}) \\ - (\vec{A} \cdot \vec{\nabla}) \vec{B} + (\vec{B} \cdot \vec{\nabla}) \vec{A}$$

(d) (i) Write the tensor form of the angle between two lines with direction cosines l_i, m_i and also write the condition for the two lines to be co-planar. 3+3=6

(ii) Define moment of inertia tensor. Show that it is a symmetric tensor of order 2. 1+3=4

(e) (i) Find the eigenvalue and eigenvectors of the matrix

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

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(ii) If H matrix is Hermitian, and I is identity matrix, show that

$(H - iI)(H + iI)^{-1}$ is a unitary matrix.

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