

Total number of printed pages–6

3 (Sem–4/CBCS) PHY HC 1

2021

PHYSICS

(Honours)

Paper : PHY–HC–4016

(Mathematical Physics – III)

Full Marks : 60

Time : Three hours

***The figures in the margin indicate
full marks for the questions.***

GROUP–A

(Full Marks : 30)

1. Answer the following questions : $1 \times 5 = 5$
 - (a) Write the polar form of a complex number.
 - (b) State Convolution theorem on Fourier transform.

Contd.

- (c) Prove $L(1) = \frac{1}{s}$, $s > 0$, using the definition of Laplace transform.
- (d) Find the Residue of the complex function $f(z) = \frac{1}{1+z^2}$ at the pole $z = i$.
- (e) Write the law of transformation for the tensor A_n^{lm} .

2. Answer the following questions : $2 \times 5 = 10$

- (a) Find the modulus and argument of $-3i$.
- (b) Check whether the function $f(Z) = \operatorname{Re} Z$ is analytic or not.
- (c) Write the Fourier's sine and cosine transform.
- (d) If $L[f(x)] = \bar{f}(s)$, then show that $L[e^{ax} f(x)] = \bar{f}(s-a)$.
- (e) Prove that a symmetric tensor of rank 2 in N dimensional space has $\frac{N(N+1)}{2}$ independent elements.

3. Answer **any three** of the following questions :

5×3=15

(a) Prove the Cauchy's integral formula —

$$f(a) = \frac{1}{2\pi i} \int_C \frac{f(z)}{z-a} dz$$

(b) Using Cauchy's residue theorem, show that —

$$\int_0^\infty \frac{dx}{1+x^2} = \frac{\pi}{2}$$

(c) Find the Fourier transform of the function —

$$f(x) = \begin{cases} 1, & \text{for } |x| < a \\ 0, & \text{for } |x| > a \end{cases}$$

(d) Find the Laplace transform of $f(t)$, where,

$$f(t) = \begin{cases} \sin t, & 0 < t < \pi \\ 0, & t > \pi \end{cases}$$

(e) What is Kronecker delta ? Show that it is a mixed tensor of rank 2.

GROUP-B

(Full Marks : 30)

4. Answer **any three** of the following questions :

(a) (i) Obtain the Cauchy-Reimann conditions for the function $f(z) = u + iv$ to be analytic, in a domain, u and v are functions of x and y . Are the conditions sufficient ? 6+1 =7

(ii) Expand the following function in Taylor series :

$$f(z) = \frac{1}{z+1}, \text{ about } z = 1. \quad 3$$

(b) Evaluate the following integrals using calculus of residues : 5×2=10

(i)
$$\int_{-\infty}^{+\infty} \frac{\sin x}{x} dx$$

(ii)
$$\int_0^{2\pi} \frac{d\theta}{5 - 4 \sin \theta}$$

- (c) Using Fourier transform, solve the one-dimensional heat flow equation

$$\frac{\partial u(x,t)}{\partial t} = \frac{\partial^2 u(x,t)}{\partial x^2}, \quad x > 0, t > 0, \text{ subject to}$$

the conditions –

(i) $u(0,t) = 0$

(ii) $u(x,0) = \begin{cases} 1, & 0 < x < 1 \\ 0, & x \geq 1 \end{cases}$

(iii) $u(x,t)$ is bounded. 10

- (d) Evaluate the following inverse Laplace transforms – 4+6=10

(i) $L^{-1} \left[\frac{s^2}{(s+1)^2} \right]$

(ii) $L^{-1} \left[\frac{4P+5}{(P-4)^2(P+3)} \right]$

- (e) (i) What is Levi-Civita tensor ? Prove that $\delta_{ij} \epsilon_{ijk} = 0$. 2+1=3

(ii) Prove that

$$\vec{A} \cdot \vec{B} = \sum_{i=1}^3 \sum_{j=1}^3 A_i B_j \delta_{ij}. \quad 3$$

- (iii) Prove that tensor of rank 2 could be written as a sum of symmetric and asymmetric tensor. 4
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