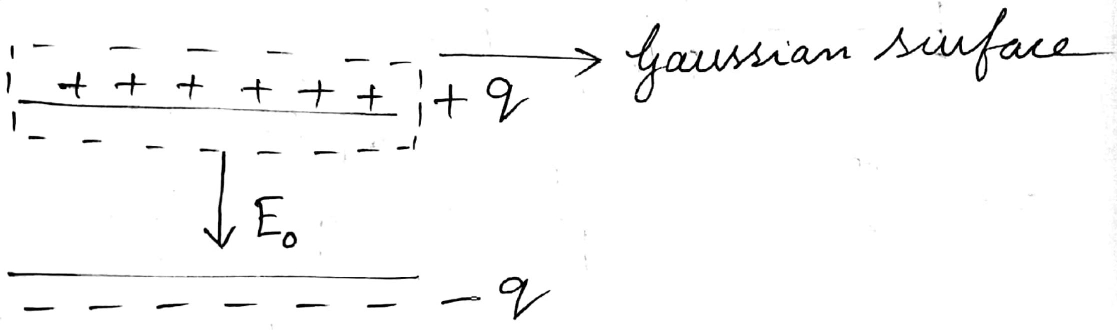
Gauss's law in dielectric medium:-

Let us consider parallel plate capacitor filled with a dielectric medium. When no dielectric medium is present, the electric field E_0 at any point on the Gaussian surface is given by,

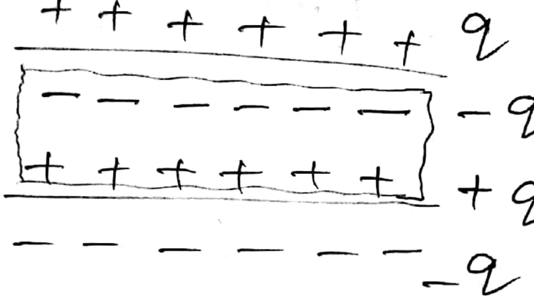


$$\int E_0 \cdot ds = \frac{q}{\epsilon_0}$$

$$\Rightarrow E_0 A = \frac{q}{\epsilon_0}$$ (where A is the area of the plate)

$$\therefore E_0 = \frac{q}{\epsilon_0 A}$$

When dielectric medium is placed, the net charge within the Gaussian surface is $q - q'$, where q' is the induced surface charge.



The electric field E is given by,

$$\int E.ds = \frac{q-q'}{\epsilon_0}$$

$$\Rightarrow EA = \frac{q-q'}{\epsilon_0} \Rightarrow E = \frac{q}{\epsilon_0 A} - \frac{q'}{\epsilon_0 A} \longrightarrow (1)$$

$$\therefore E = E_0 - \frac{q'}{\epsilon_0 A} \longrightarrow (2)$$

If V_d and V_0 are the potential differences with and without the dielectric respectively, then $\frac{E_0}{E} = \frac{V_0}{V_d} = K$ (dielectric constant)

$$\therefore E = \frac{E_0}{K} = \frac{q}{\epsilon_0 A K} \longrightarrow (3)$$

Putting (3) in (1), we get,

$$\frac{q}{\epsilon_0 A K} = \frac{q}{\epsilon_0 A} - \frac{q'}{\epsilon_0 A}$$

$$\therefore q' = q\left(1 - \frac{1}{K}\right) \longrightarrow (4)$$

This shows that the induced surface charge q' is always less than the free charge q

and is zero, when $K = 1$. (i.e. in absence of dielectric)

$$\therefore \oint E.ds = \frac{q - q'}{\epsilon_0}$$

$$= \frac{1}{\epsilon_0}\left[q - \left\{q\left(1 - \frac{1}{K}\right)\right\}\right]$$

$$= \frac{1}{\epsilon_0}\frac{q}{K}$$

$$\therefore \boxed{\epsilon_0 \oint K E.ds = q} \longrightarrow (5)$$

Equation (5) represents Gauss's law in presence of dielectric.

Here the flux integral contains a factor K and the effect of the induced charge is ignored by taking into account dielectric constant (K).

From equation (4), we get

$$q' = q\left(1 - \frac{1}{K}\right)$$

$$q' = q - \frac{q}{K}$$

$$\therefore q = \frac{q}{K} + q'$$

$$\therefore \frac{q}{A} = \frac{\epsilon_0 q}{\epsilon_0 KA} + \frac{q'}{A}$$

The term $\frac{q'}{A}$ is the induced surface charge per unit area and is known as the electric polarisation $\vec{P}$.

$$\therefore \frac{q}{A} = \epsilon_0 \left(\frac{q}{\epsilon_0 KA} \right) + \vec{P} \longrightarrow (6)$$

Here, $\frac{q}{\epsilon_0 KA}$ is the electric field intensity within the dielectric. $\epsilon_0 \int KE.ds = q$ or

$$\epsilon_0 KEA = q \Rightarrow E = \frac{q}{\epsilon_0 KA}.$$

From equation (6), we can write,

$$\frac{q}{A} = \epsilon_0 E + P$$

The term $\epsilon_0 E + P$ is called electric displacement D.

$$\therefore \vec{D} = \epsilon_0 \vec{E} + \vec{P}$$

Gauss's law in presence of dielectric can be written as,

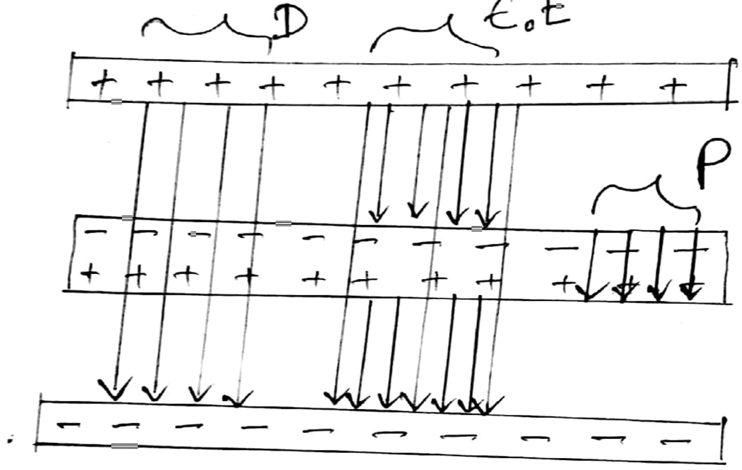
$$\epsilon_0 \int E.ds = q - q'$$

$$\Rightarrow \epsilon_0 \int E.ds = q - \int \vec{P}.ds$$

$$\Rightarrow \int (\epsilon_0 E + P).ds = q$$

$$\Rightarrow \boxed{\int \vec{D}.ds = q}$$

Susceptibility χ and K :-



The electric displacement vector $\vec{D}$ is connected with the free charge only. These lines begin and end on the free charges. The flux of D equals the free charge.

The polarization vector $\vec{P}$ is connected with the induced surface charge. These lines begin and end on the induced (polar[ized]) charges.

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The electric field intensity E is connected with the free and induced (bound) charges. $\vec{E}$ depends upon the presence of both kinds of charges.

In free space, where no dielectric is present $P = 0$

$$\therefore \; D = \epsilon_0 E$$

In the presence of a dielectric, D and P both can be expressed in terms of E.

$$\therefore \; D = \frac{q}{A} = \frac{K\epsilon_0 q}{K\epsilon_0 A} = K\epsilon_0 E \qquad \left(E = \frac{q}{\epsilon_0 K A}\right)$$

$$\therefore \; \boxed{D = K\epsilon_0 E}$$

The term $\epsilon_0 K$ is called the permittivity ϵ of the medium,

$$\boxed{D = \epsilon E} \qquad (\epsilon = \epsilon_0 K)$$

In vaccum, $K = 1$, $\epsilon = \epsilon_0$

Here, ϵ_0 is known as the permittivity of a vaccum or free space.

$$\vec{D} = \epsilon_0 \vec{E} + \vec{P}$$

$$\vec{D} = K\epsilon_0 \vec{E}$$

$$\epsilon_0 \vec{E} + \vec{P} = K\epsilon_0 \vec{E}$$

$$\therefore \vec{P} = \epsilon_0 (K-1) E$$

$\chi = K - 1$ is called the susceptibility of the dielectric medium.

$$\boxed{P = \epsilon_0 \chi E}$$

$$\boxed{\chi = K - 1 = \frac{\epsilon}{\epsilon_0} - 1}$$