Stationary or Standing Wave:

8 Stationary waves are formed in any
9 medium when two exactly similar wave
10 trains having the same wavelength,
11 velocity and amplitude travel along
12 the same straight line in a medium
1 in opposite direction.

2 Let the two waves be represented
3 by,

$$y_1 = a \sin \frac{2\pi}{\lambda}(ct - x)$$

$$y_2 = a \sin \frac{2\pi}{\lambda}(ct + x)$$

Resultant,

$$y = y_1 + y_2$$

NOTES

$$y = a \sin \frac{2\pi}{\lambda}(ct - x) + a \sin \frac{2\pi}{\lambda}(ct + x)$$

$$= 2a \sin \frac{2\pi}{\lambda} ct . \cos \frac{2\pi}{\lambda} x$$

The Slope of the curve or strain,

$$\frac{dy}{dx} = -\frac{4\pi a}{\lambda} \sin \frac{2\pi}{\lambda} ct \sin \frac{2\pi}{\lambda} x$$

The velocity of the Vibrating Particles

$$\frac{dy}{dt} = \frac{4\pi a}{\lambda} \cos \frac{2\pi}{\lambda} ct \cos \frac{2\pi}{\lambda} x$$

When, $\cos \frac{2\pi}{\lambda} x = 0$ gives $y = 0$, $\frac{dy}{dt} = 0$, $\frac{dy}{dx}$ is maximum at places given by

$$\frac{2\pi}{\lambda} x = (2m+1)\frac{\pi}{2},$$ where m is an integer,

or $$x = \left(\frac{m}{2} + \frac{1}{4}\right)\lambda$$

Thus there are points at regular intervals which are never displaced. They are called Nodes. The distance between ~~two~~ two consecutive nodes is $\frac{\lambda}{2}$

NOTES

The displacement will be maximum or minimum at places given by

$$\cos \frac{2\pi}{\lambda} x = \pm 1$$

$$\frac{2\pi}{\lambda} x = m\pi$$

$$\therefore x = \frac{m\lambda}{2}$$ (m is zero or integer)

At any instant,

$$\sin \frac{2\pi}{\lambda} ct = \pm 1$$

$$\therefore t = (2m+1)\frac{\pi}{4}$$

Thus there are points at regular intervals where the particles periodically attain their maximum value 2a or a minimum value $-2a$ where strain $\frac{dy}{dx} = 0$ and velocity $\frac{dy}{dt}$ is maximum. These are called Antinodes. The distance between two consecutive antinodes is

$\frac{\lambda}{2}$ and between a node and next antinode is $\frac{\lambda}{4}$.

—— x ——

Transverse Vibration of Stretched String

54. Frequency of a vibrating string

It is seen that velocity c of the transverse vibration along a string is given by $c=\sqrt{\frac{T}{\rho}}$. T and ρ being the tension and line density of the string respectively. But $c=n\lambda$, λ is the wavelength and n is the frequency of vibration. So

$$n=\frac{c}{\lambda}=\frac{1}{\lambda}\sqrt{\frac{T}{\rho}} \qquad \text{...(1)}$$

when the string vibrates, stationary waves are set up as being reflected from two fixed ends. The fixed ends being always nodes and in between these two nodes there is an antinode when the string vibrates with only one loop, *i.e.* two fixed ends are two nodes with only one antinode in the middle, l=the length of the string is given by

$$l=\frac{\lambda}{2} \text{ or } n=\frac{1}{2l}\sqrt{\frac{T}{\rho}} \qquad \text{...(2)}$$

The string can vibrate in different modes, within the length of the string, two, three, four etc. number of loops may be formed, keeping always two fixed ends, nodes, as shown in Fig. 39.

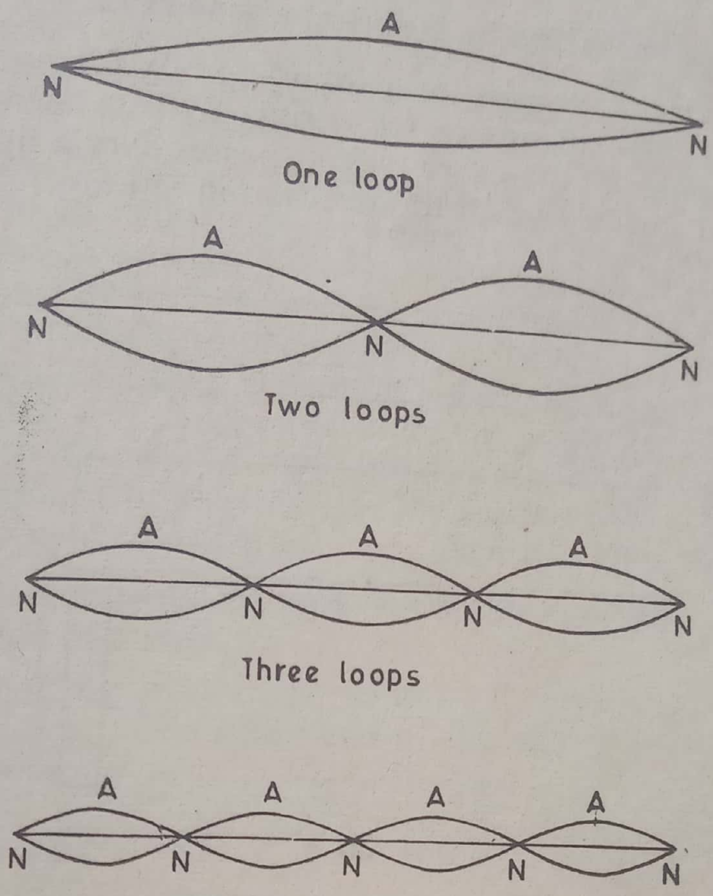


Fig. 39

So vibration with

One loop $\quad \lambda_1 = 2l, \quad n_1 = \frac{1}{2l}\sqrt{\frac{T}{\rho}}$ ...(3·1)

Two loops $\quad \lambda_2 = \frac{2l}{2} \quad n_2 = \frac{2}{2l}\sqrt{\frac{T}{\rho}}$ ...(3·2)

Three loops $\quad \lambda_3 = \frac{2l}{3} \quad n_3 = \frac{3}{2l}\sqrt{\frac{T}{\rho}}$ ...(3·3)

s loops $\quad \lambda_s = \frac{2l}{s} \quad n_s = \frac{s}{2l}\sqrt{\frac{T}{\rho}}$ ...(4)

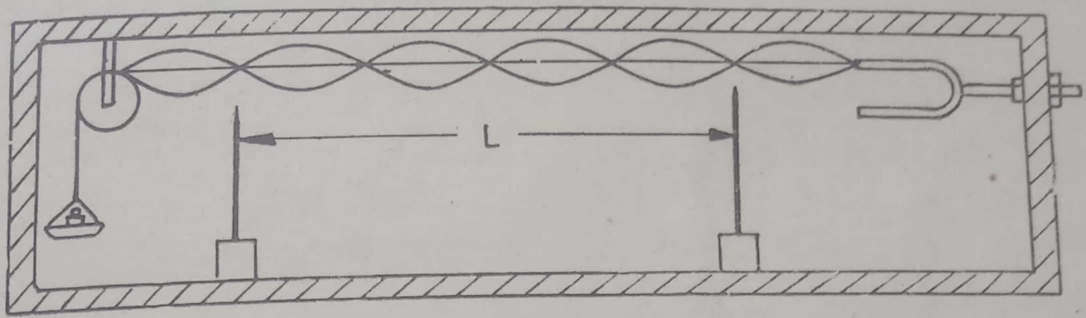
Thus it is found that the frequency of the gravest role or fundamental n_1 is inversely proportional to the length of the vibrating string (*ii*) directly proportional to the square root of the tension to which the string is subjected (*iii*) inversely proportional to the square root of the linear density of the string. These are laws of transversely vibrating string. If r be the radius of the string and D be density of the material of the string then

$$\pi r^2 D = \rho$$

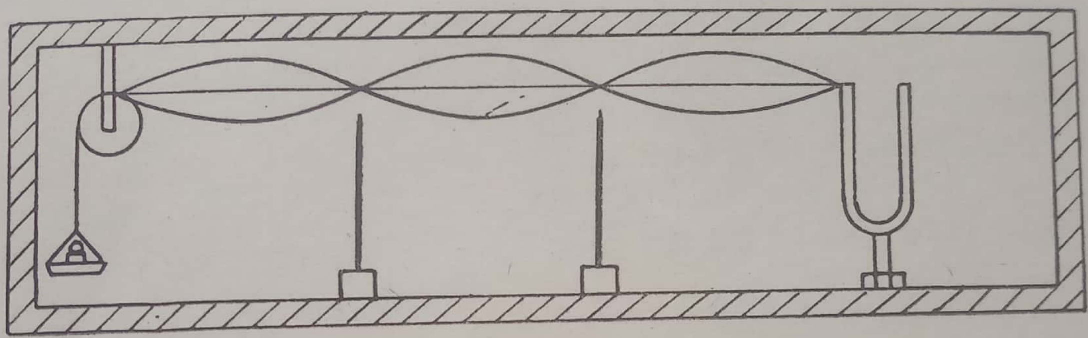
So the frequency of the note emitted by the vibrating string is inversely proportional to the radius of the string and inversely proportional to the square root of the density of the material of the string.

57. Meldes Experiment and verification of the laws of transverse vibration of a string.

The string is attached to one of the prong and passes over a smooth pulley. The other end of the strings carries the scale pan



(*a*)



(*b*)

Fig. 41

on which weights are placed. When the fork is excited, the string is set in stationary vibration due to reflection of waves from other end, hence loops are formed which are visible in the string.

Transverse arrangement, Fig. 41(*a*). Here the string vibrates up and down with the prong of the fork. As stationary waves are formed due to reflection from the other end fixed, loops with nodes and antinodes are formed. The distance between the ends of a definite number N of loops is located by placing pointed stands at two extreme nodes and the length L is measured, which gives L/N, the length of a single loop between two consecutive nodes. Let l be the average length of a loop then $n=\frac{1}{2l}\sqrt{\frac{T}{\rho}}$ where n is the frequency of vibration, T the tension (which is the weight of the scale pan with standard weights placed on it (expressed in dynes), ρ the mass per unit length of the string obtained by weighing a definite length of the same sample of the string.

The nodes, hence the loops are made pronounced by adjusting the tension or the length of the string. In this case the frequency of the fork is equal to the frequency of the string.

(*ii*) **Longitudinal arrangement,** Fig. 41 (*b*). In this case the string is tightened, straight when the prong, during its vibration, is away from the pulley. The string is sufficiently loose when the prong comes nearer to the pulley. Thus the string makes quarter of its vibration during the time the prong makes half its vibration. Thus in this case the frequency of the string is half the frequency of the fork. If the mean length of a loop is l then

$$\frac{n}{2}=\frac{1}{2l}\sqrt{\frac{T}{\rho}} \text{ or } n=\frac{1}{l}\sqrt{\frac{T}{\rho}},$$